647510 **BSc3MatC0301** **Seat No :________**

B.Sc. Semester - 3 (*CBCS*) Examination
Nov/Dec. -2021 (OLD COURSE)
MATHEMATICS (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q-1 (A) Answer any one: **(07)**

(1) If $\lim_{n\to\infty} a_n = a$ then prove that:

$$\lim_{n\to\infty}\left(\frac{a_1+a_2+\cdots a_n}{n}\right) = l.$$

(2) Prove that: $\lim_{n\to\infty}\sqrt[n]{a} = 1\,; a > 0.$

(B) Answer any one: **(04)**

(1) Discuss the convergence of the sequence, where $S_1 = \sqrt{6}, S_{n+1} = \sqrt{6S_n},\ n \in N$ and find its limit, if exists.

(2) Show that $\lim_{n\to\infty}\left[\frac{1}{\sqrt{n}} + \frac{1}{\sqrt{n+1}} + \cdots + \frac{1}{\sqrt{2n}}\right]$ does not exist.

(C) Answer any one: **(03)**

(1) Define: Oscillate sequence and show that the sequence $\left\{(-1)^n\left(1+\frac{1}{n}\right)\right\}$ is oscillates finitely.

(2) Prove that every convergent sequence is bounded.

Q-2 (A) Answer any one: **(07)**

(1) State Raabe's test. Discuss the convergence of $\sum \frac{1\cdot3\cdot5\cdots\cdots(2n-1)}{2\cdot4\cdot6\cdots\cdots(2n)}x^n.$

(2) State: D'Alembert's ratio test and discuss the convergence of

$$\frac{\sqrt{2}-1}{1} + \frac{\sqrt{3}-\sqrt{2}}{2} + \frac{\sqrt{4}-\sqrt{3}}{3} + \cdots$$

(B) Answer any one: **(04)**

(1) Find the radius of convergence and interval of convergence of the series $2 - \frac{4}{5}x + \frac{6}{5^2}x^2 - \frac{8}{5^3}x^3 + \cdots$

(2) Define: Geometric series and test the convergence of series $\sum_{n=0}^{\infty}\frac{3^{2n}}{2^{3n}}$.

(C) Answer any one: **(03)**

(1) Show that the series $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \cdots$ is convergent but not absolutely convergent.

(2) Show that the series $\sum(-1)^n$ oscillates finitely.

Q-3 (A) Answer any one: **(07)**

(1) If $\bar{f}$ and $\bar{g}$ are vector functions defined on the domain D whose first order derivatives exist and $\emptyset$ is a scalar function on D whose first order partial derivative also exist then prove that:

(i) $div(\emptyset\bar{f}) = (grad\emptyset)\cdot\bar{f} + \emptyset div\bar{f}$

(ii) $div(\bar{f}\times\bar{g}) = \bar{g}\cdot curl\bar{f} - \bar{f}\cdot curl\bar{g}$

(2) Define: Irrotational function. If $\bar{f}$ and $\bar{g}$ are irrotational functions on D then show that $\bar{f}\times\bar{g}$ is a solenoidal function.

(B) Answer any one: **(04)**

(1) Prove: $\nabla^2(logr) = \frac{1}{r^2}$, where $r = |\bar{r}| = \sqrt{x^2+y^2+z^2}$.

(2) Define: Solenoidal function. If

$$\bar{f} = (2x+3y+az)\hat{\imath} + (bx+2y+3z)\hat{\jmath} + (2x+cy+3z)\hat{k}$$

irrotational function then find a, b and c.

(C) Answer any one: **(03)**

(1) Find the unit normal at the surface $\emptyset(x, y, z) = x^2y + y^2x + z^2$ at the point $(2, -2, 3)$.

(2) If $\bar{f} = x^2y\hat{\imath} - 2xz\hat{\jmath} + 2yz\hat{k}$ then find (i) $curl\bar{f}$ (ii) $div\bar{f}$.

Q-4 (A) Answer any one: **(07)**

(1) Prove that :

$$\iint\limits_{D} \sqrt{\frac{a^2b^2 - b^2x^2 - a^2y^2}{a^2b^2 + b^2x^2 + a^2y^2}}\, dxdy = \frac{\pi}{2}(\pi - 2)ab\ ;$$

where D is ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(2) Define: Triple integral and evaluate:

$\iiint\limits_{R} (x + y + z)^2 dxdydz$, where R is surface bounded by

$x \geq 0, y \geq 0, z \geq 0$ and $x + y + z \leq 1$.

(B) Answer any one: **(04)**

(1) Evaluate: $\iint\limits_{R} x\cos(x + y)dxdy$, where R is triangle whose vertices are $(0,0), (\pi, 0)$ and (π, π).

(2) Prove: $\iiint\limits_{R} (x^2 + y^2 + z^2)^5 dxdydz = \frac{4\pi a^{13}}{13}$

Where R is sphere $x^2 + y^2 + z^2 = a^2$.

(C) Answer any one: **(03)**

(1) Evaluate: $\int_0^1 \int_0^x e^{\frac{y}{x}}\, dydx$.

(2) Change in cylindrical co-ordinates

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{1+x+y} (x^2 - y^2)\, dzdydx$$

Q-5 (A) Answer any one:

(1) State and prove Green's theorem for a plane.

(2) In usual notation prove that:

$\beta(p, q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$, where $p > 0, q > 0$.

(B) Answer any one: **(04)**

(1) If $\bar{V} = (x + z)\hat{\imath} + (y + z)\bar{\jmath} + (x + y)\bar{k}$ and S is a Surface of a sphere $x^2 + y^2 + z^2 = 16$ then find $\iint\limits_{S} \bar{V} \cdot \bar{n}\, d\sigma$.

(2) Prove: $\int_{(1,0)}^{(x,y)} \frac{1+y^2}{x^3} dx - \frac{1+x^2}{x^2} ydy$ is independent of path and find its value.

(C) Answer any one: **(03)**

(1) Evaluate: $\int\limits_{C} \bar{F} \cdot d\bar{r}$, where C is line segment joining (2,0) and (3,2) and $\bar{F} = (x^2 + y)\hat{\imath} + (2x + y)\hat{\jmath}$.

(2) In usual notation, prove that: $\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right) = \sqrt{2}\pi$.

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